Comparing Networks by their Group Structure
with an application to acculturation networks

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suppose we have several personal networks . . .

. . . and want to determine their differences and similarities

- number of actors, ties, or connected components, degree distribution, diameter
- reciprocity, transitivity, clustering coefficient
- isomorphic, edit distance, common subgraphs

this talk: compare networks by their group structure
Overview
Comparing Networks by their Group Structure

1. example application: towards a network measure for acculturation
2. defining class-level networks
3. average class-level networks
4. conclusion and future work
Acculturation

traditional usage: outcome of cultures coming into contact

recent usage: measuring the level of integration of migrants into a host culture
Traditional Acculturation Scale (ARSMA II)
[Cuéllar/Arnold/Maldonado’95] modes of acculturation [Berry’97]

- I speak English
- I speak Spanish
- I associate with Anglos
- I associate with Mexicans
- I enjoy English language TV
- I enjoy Spanish language TV
- My friends now are of Anglo origin
- My friends now are of Mexican origin
- ...

▶ influence of personal networks?
Traditional Acculturation Scale (ARSMA II)  
[Cuéllar/Arnold/Maldonado’95]  
modes of acculturation [Berry’97]

- I speak English  
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- ...  

Influence of personal networks?
Data Set www.egoredes.net

approximately 500 respondents (egos):

1. **(questions about ego)** age, years of residence, health, ARSMA II, . . .
2. **(name generator)** list of 45 alters
3. **(questions about alters)** born, lives, type of relation, . . .
4. **(ties)** which alters know each other

**this talk**: differences and similarities between these networks (comparison on the **class level**)

special thanks for providing the data goes to the project **Acculturation & Personal Networks across cultures**

⇒ see talks by Javier Avila, Miranda Lubbers, Chris McCarty, and José Luis Molina on Saturday
Overview
Comparing Networks by their Group Structure

1. example application:
   towards a network measure for acculturation

2. **defining class-level networks**

3. average class-level networks

4. conclusion and future work
two steps

1. defining actor classes, dependent on
   - network structure (e.g., structural/regular equivalence)
   - actor attributes (yields labeled classes)

   classes become nodes

2. defining inter-class ties
   (how strongly are two classes connected?)
two steps

1. defining actor classes, dependent on
   - network structure (e.g., structural/regular equivalence)
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   classes become nodes

2. defining inter-class ties
   (how strongly are two classes connected?)
defining actor classes
argentinean woman living in spain

considering country of origin and country of residence (4 selected classes)
(origin) Argentineans living in Argentina
(fellows) Argentineans living in Spain
(host) Spaniards living in Spain
defining actor classes
argentinean woman living in spain

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(origin) Argentineans living in Argentina
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(host) Spaniards living in Spain
(all other cases)
defining actor classes
argentinean woman living in spain

considering *country of origin* and *country of residence*
(4 selected classes)
*(origin)* Argentineans living in Argentina
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*(host)* Spaniards living in Spain
*(transnationals)* all other cases
defining actor classes
argentinean woman living in spain

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actor classes become nodes

class-level network

ditionals

edge weights?
actor classes become nodes

class-level network

edge weights?
Normalization of Edge-Weights
how strongly are two classes connected?

given: network $G = (V, E)$ and two actor classes $A$ and $B$

1. (un-normalized count) $\# \{(a, b) \in E ; a \in A, b \in B\}$
   $\Rightarrow$ larger classes will be stronger connected

2. (density) $\frac{\#\{(a,b)\in E ; a\in A b\in B\}}{|A|\cdot|B|}$
   $\Rightarrow$ tends to zero when class sizes increase (sparsity)

3. (avg. number of $B$-neighbors) $\frac{\#\{(a,b)\in E ; a\in A b\in B\}}{|A|}$
   $\Rightarrow$ asymmetric class-level network

4. (symmetric normalization) $\frac{\#\{(a,b)\in E ; a\in A b\in B\}}{\sqrt{|A|\cdot|B|}}$
   $\Rightarrow$ this is what we take
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Class-Level Network

argentinean woman living in spain

node size = class size
darkness = intra-class ties
79 Argentinean Migrants to Spain
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do Argentineans have different networks than Moroccans?

**arithmetic mean** \( \bar{X} = \frac{X_1 + X_2 + \cdots + X_N}{N} \)

**how to add networks?**

- add class-sizes
- add un-normalized edge counts
- normalize at the end
Average Networks of …

…migrants to Spain

Dominican Republic  Argentina  Equatorial Guinea  Senegal/Gambia  Morocco

Dominican Republic  Colombia  Cuba  Haiti  Mexico  Puerto Rico

…migrants to the USA
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country **less than 1 year ago**
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 1 year ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 2 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 3 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 4 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 5 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 6 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 7 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 8 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 9 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 10 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country $11$ years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 12 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 13 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 14 years ago
Dependence on Time of Residence

averaging over all migrants that . . .  
. . . moved to host country 15 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 16 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 17 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 18 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 19 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 20 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country **21** years ago
Dependence on Time of Residence

averaging over all migrants that …

… moved to host country 22 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 23 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country **24** years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 25 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 26 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 27 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 28 years ago
Dependence on Time of Residence

averaging over all migrants that . . . 
. . . moved to host country 29 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 30 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 31 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 32 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 33 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country **34** years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 35 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 36 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 37 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 38 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 39 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 40 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 41 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 42 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 43 years ago
Dependence on Time of Residence

averaging over all migrants that . . .

. . . moved to host country 44 years ago
Dependence on Time of Residence

averaging over all migrants that . . .
. . . moved to host country 45 years ago
Dependence on Time of Residence

averaging over all migrants that ...

... moved to host country 46 years ago
Conclusion and Future Work

- comparing networks by their group structure
  (actor classes defined by attributes)
  - very simple and efficient
  - allows averaging over sets of networks
  - needs meaningful definition of actor classes

- future work
  - acculturation mode $\leftrightarrow$ network structure
  - consider classes defined by attributes and ties
    (e.g., relative regular equivalence [Boyd/Everett’99])
  - determine meaningful combinations of attributes